NATIONAL SENIOR CERTIFICATE

GRADE 12

## SEPTEMBER 2016

## MATHEMATICS P2

MARKS: 150

TIME: $\quad 3$ hours


This question paper consists of 15 pages including 1 information sheet, and a SPECIAL ANSWER BOOK.

## INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary round off your answers to TWO decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

## QUESTION 1

A Grade 12 learner recorded the daily weight of a bar of soap after he had taken his shower in the morning. The table below shows the data he recorded: "Day" shows the number of days since the beginning of the experiment and "Weight" shows the weight of the bar of soap in grams.

| Day | 0 | 1 | 4 | 6 | 9 | 12 | 17 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight | 124 | 121 | 103 | 90 | 71 | 50 | 27 | 16 |

### 1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK.

### 1.2 Calculate an equation for the least squares regression line for the data.

1.3 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1.
1.4 Determine on which morning the mass of the bar of soap will be less than 80 grams.
1.5 Calculate the value of the correlation coefficient. Round off your answer to THREE decimal
places.
1.6 Comment on the strength of the relationship between the variables.

## QUESTION 2

The stem-and-leaf plot shows how many pages of a textbook learners in a Mathematics class revised before writing their examination.

| 5 | 7 |
| :--- | :--- |
| 4 | 344 |
| 3 | 111468 |
| 2 | 24899 |
| 1 | 5567 |
| 0 | 35 |

2.1 How many learners were in the class?
2.2 What was the least number of pages of revision completed?
2.3 Calculate the mean of the given data.
2.4 Determine the standard deviation of the given data.
2.5 Calculate the percentage of data that lies outside ONE standard deviation from the mean.
Show ALL your calculations.

## QUESTION 3

In the diagram below $\mathrm{P}(-4 ; 1), \mathrm{Q}(1 ;-4), \mathrm{R}(4 ; 5)$ and S are the vertices of quadrilateral PQRS . $\mathrm{SQ} \perp \mathrm{PR}$ with T on PR . T is the midpoint of line SQ .

3.1 Determine the gradient of PR.
3.2 Hence, determine the equation of line SQ.
3.3 Show that the coordinates of T are $(-2 ; 2)$.
3.4 Hence, determine the coordinates of S.
3.5 Calculate the area of $\triangle \mathrm{PQS}$.

## QUESTION 4

$x^{2}+y^{2}+8 x-6 y=-5$, is the equation of the circle with centre $M$. UE is a tangent to the circle at Q. QMD, DA, AU and UQE are straight lines. DU is parallel to the $x$-axis.

4.1 Determine the coordinates of M , the centre of the circle.
4.2 Calculate the coordinates of Q , if $y<2$.
4.3 Calculate the equation of tangent UE.
4.4 Write down the equation of DU .
4.5 Calculate the coordinates of $U$.
4.6 Prove that QUAD is a cyclic quadrilateral.

## QUESTION 5

5.1 If $\cos 52^{\circ}=t$, determine, in its simplest form the following in terms of $t$, WITHOUT the use of a calculator.

$$
\begin{equation*}
\text { 5.1.1 } \sin \left(-52^{\circ}\right) \tag{3}
\end{equation*}
$$

5.1.2 $\cos 19^{\circ}$
5.2 Simplify WITHOUT the use of a calculator:

$$
\begin{equation*}
\frac{2 \cos \left(180^{\circ}+x\right) \cdot \sin \left(180^{\circ}-x\right) \cdot \sin 74^{\circ}}{\sin \left(x+360^{\circ}\right) \cdot \sin 37^{\circ} \cdot \sin 53^{\circ} \cdot \sin \left(x-90^{\circ}\right)} \tag{7}
\end{equation*}
$$

5.3 Given: $\frac{2 \sin x}{2(1-\cos 2 \mathrm{x})}$
5.3.1 Calculate all the values of $x$ for which the expression above is undefined.
5.3.2 Prove that $\frac{2 \sin x}{2(1-\cos 2 \mathrm{x})}=\frac{1}{\sin x}$

## QUESTION 6

Given: $f(x)=\cos 2 x$ and $g(x)=\sin \left(x+60^{\circ}\right)$ for $x \in\left[-90^{\circ} ; 180^{\circ}\right]$.
6.1 Solve for $x$ if $f(x)=g(x)$ and $x \in\left[-90^{\circ} ; 180^{\circ}\right]$.
6.2 Sketch the graph of $f$ and $g$ on the same set of axes for $x \in\left[-90^{\circ} ; 180^{\circ}\right]$. Clearly show ALL intercepts with the axes, points of intersection as well as turning points.
6.3 Write down the period of $g\left(\frac{3}{2} x\right)$.
6.4 Determine $h$ if $h(x)=f\left(x-45^{\circ}\right)-1$.

## QUESTION 7

The figure shows an open birthday card. The length of the card is 145 mm and the breadth is 103 mm . The card is placed such that the angle formed between the two sides is $120^{\circ}$.

7.1 Calculate the length of NP.
7.2 Calculate the length of PQ .
7.3 Determine the size of $P \widehat{N} Q$.

## Give reasons for ALL statements in QUESTION 8, 9, 10 and 11.

## QUESTION 8

8.1 Complete the following statement: A line drawn from the centre of the circle perpendicular to a chord ...
8.2 In the diagram below O is the centre of circle PMQNF. PN and FM are diameters.

QN || FM
$\mathrm{OL}=3 \mathrm{ML}$
$\mathrm{QP}=14$ units

8.2.1 Prove that L is the midpoint of QP.
8.2.2 Write down the length of MF in terms of ML.
8.2.3 Determine the length of ML. Leave your answer in surd form.

## QUESTION 9

In the diagram below ABCD is a cyclic quadrilateral. RBP is a tangent to the circle with centre O . $\widehat{B_{2}}=30^{\circ}$ and $\widehat{B_{6}}=70^{\circ}$.


Determine the size of each of the following angles:
$9.1 \widehat{O_{1}}$
$9.2 \hat{A}$
$9.3 \hat{C}$
$9.4 \quad A \widehat{D} B$

## QUESTION 10

10.1 In the diagram below O is the centre of circle UYZ.

XUT is a tangent to the circle at U .


Prove that $X \widehat{U} Z=\hat{Y}$.
10.2 CS and AS are two tangents of circle ABC . AB is produced to R . AC and SR are produced to $\mathrm{T} . \mathrm{AB}=\mathrm{BC}$ and $\mathrm{TS} \| \mathrm{BC}$. Let $\widehat{C_{1}}=x$.

10.2.1 Name with reasons, 5 other angles each equal to $x$.
10.2.2 Hence, show that $\triangle \mathrm{SCT}$ is an isosceles triangle.
10.2.3 If it is further given that, $\mathrm{CS}=4 \mathrm{~cm}, \frac{A R}{B R}=\frac{3}{2}$, calculate the length of AT.

## QUESTION 11

FAN is a common tangent to the smaller circle ABCD and the larger circle ARZP. FP is a tangent to the smaller circle at C . The straight line ABR meets the larger circle at R .

11.1 Prove that BC $\|$ RZ.
11.2 Hence, prove that BC is a tangent to circle ACP .
11.3 Prove that $\triangle$ RZA ||| $\Delta \mathrm{DPC}$.
11.4 Hence, show that $\frac{D C}{C P} \times \frac{A C}{A B}=1$.

## INFORMATION SHEET MATHEMATICS

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

$$
\begin{aligned}
& A=P(1+n i) \\
& A=P(1-n i) \\
& A=P(1-i)^{n} \\
& A=P(1+i)^{n} \\
& \sum_{i=1}^{n} 1=n \quad \sum_{i=1}^{n} i=\frac{n(n+1)}{2} \quad T_{n}=a+(n-1) d \\
& \mathrm{~S}_{n}=\frac{n}{2}(2 a+(n-1) d) \\
& T_{n}=a r^{n-1} \\
& S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; \quad r \neq 1 \\
& S_{\infty}=\frac{a}{1-r} ;-1<r<1 \\
& F=\frac{x\left[(1+i)^{n}-1\right]}{i} \\
& P=\frac{x\left[1-(1+i)^{-n}\right]}{i} \\
& f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \\
& d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& \mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right) \\
& y=m x+c \\
& y-y_{1}=m\left(x-x_{1}\right) \\
& m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta \\
& (x-a)^{2}+(y-b)^{2}=r^{2} \\
& \text { In } \triangle A B C: \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \quad \text { area } \triangle A B C=\frac{1}{2} a b \cdot \sin C
\end{aligned}
$$

$\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta$
$\cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta$
$\sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta$
$\cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta$
$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array} \quad \sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha\right.$

$$
\bar{x}=\frac{\sum x}{n}
$$

$$
\sigma^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}
$$

$$
P(A)=\frac{n(A)}{n(S)}
$$

$$
\begin{aligned}
& P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) \\
& b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}}
\end{aligned}
$$

